Computer Lab Work: Complex Analysis-I

4th Semester (MAT-4204C)

Mathematica Practical Notes

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Experiment 1: Declaring and Graphing Complex Numbers

Objective: Declaring a complex number and graphical representation. e.g., $Z_1 = 3+4i$, $Z_2 = 4-7i$.

Mathematica Code:

```
Clear[Z1, Z2];
Z1 = 3 + 4*I;
Z2 = 4 - 7*I;

(* Backward-compatible plot using ListPlot and ReIm *)
ListPlot[{ReIm[Z1], ReIm[Z2]},
 PlotStyle -> Directive[PointSize[Large], Red],
 AxesLabel -> {"Real", "Imaginary"},
 PlotLabel -> "Graphical Representation of Complex Numbers",
 PlotRange -> {{-1, 5}, {-8, 5}},
 GridLines -> Automatic]
```

Experiment 2: Algebra of Complex Numbers

Objective: Program to discuss the algebra of complex numbers. Find Z_1+Z_2, Z_1-Z_2, Z_1*Z_2, and Z_1/Z_2.

Mathematica Code:

```
(* Declaring the complex numbers *)
Z1 = 3 + 4*I;
```

```
Z2 = 4 - 7*I;

(* Performing Algebraic Operations *)
addition = Z1 + Z2
subtraction = Z1 - Z2
multiplication = Z1 * Z2
division = Z1 / Z2
```

Experiment 3: Array of Complex Numbers

Objective: To find conjugate, modulus, and phase angle of an array of complex numbers. e.g., $Z = [2 + 3i, 4 - 2i, 6 + 11i, 2 - 5i]$.

Mathematica Code:

```
(* Declaring the array of complex numbers *)
Z = {2 + 3*I, 4 - 2*I, 6 + 11*I, 2 - 5*I};

(* Finding Conjugate, Modulus, and Phase Angle *)
conjugates = Conjugate[Z]
moduli = Abs[Z]
phases = Arg[Z] (* Phase angle in radians *)
```

Experiment 4: Integral Over a Straight Line Path

Objective: To compute the integral over a straight line path between two specified end points. e.g., $\int \sin(z)dz$ along the contour C from $-1 + i$ to $2 - i$.

Mathematica Code:

```
(* Parameterization of the straight line path: z(t) = z1 + t*(z2
   - z1) *)
z1 = -1 + I;
z2 = 2 - I;
z[t_] := z1 + t*(z2 - z1);

(* Compute the derivative of z with respect to t *)
dz = z'[t];

(* Performing the Integration *)
integral = Integrate[Sin[z[t]] * dz, {t, 0, 1}]
```

Experiment 5: Contour Integration

Objective: To perform contour integration. (i) $\int(z^2-2z+1)dz$ along $x=y^2+1; -2 \leq y \leq 2$. (ii) $\int(z^4+2z^2+1)dz$ parameterized by $x=\cos(t), y=\sin(t)$ for $0 \leq t \leq 2\pi$.

Mathematica Code:

```
(* Clear old definitions from memory *)
Clear[z1, z2];

(* Part (i): Parabolic Contour *)
z1[y_] := (y^2 + 1) + I*y;
integral1 = Integrate[(z1[y]^2 - 2*z1[y] + 1) * z1'[y], {y, -2,
    2}]

(* Part (ii): Circular Contour *)
z2[t_] := Cos[t] + I*Sin[t];
integral2 = Integrate[(z2[t]^4 + 2*z2[t]^2 + 1) * z2'[t], {t, 0,
    2*Pi}]
```

Experiment 6: Plotting Complex Functions

Objective: To plot the complex functions and analyze the graph. $f(z) = z, iz, z^2, z^3, e^z$ and $(z^4-1)^{1/4}$.

Mathematica Code:

```
(* 1. Clear variables *)
Clear[z, f, functions, plots, x, y];

(* 2. Define the list of functions *)
functions = {z, I*z, z^2, z^3, Exp[z], (z^4 - 1)^(1/4)};

(* 3. Workaround for ComplexPlot using DensityPlot *)
plots = Table[
  DensityPlot[
   (* Calculate the phase angle of the complex function *)
   Arg[f /. z -> x + I*y],
   {x, -2, 2}, {y, -2, 2},
   PlotPoints -> 80,
   (* Map the angle (from -Pi to Pi) to a full color wheel (0 to
      1) *)
   ColorFunction -> (Hue[(# + Pi)/(2*Pi)] &),
```

```
    ColorFunctionScaling -> False,
    Exclusions -> None,
    PlotLabel -> Style[Row[{"f(z) = ", TraditionalForm[f]}], 14,
       Bold]
  ],
  {f, functions}
]
```

Experiment 7: Taylor Series Expansion and Plotting

Objective: To perform the Taylor series expansion. (i) $f(z) = \exp(z)$ around $z = 0, n = 40$ (ii) $f(z) = \exp(z^2)$ around $z = 0, n = 160$. Plot magnitude.

Mathematica Code:

```
(* Part (i): Exp[z] around z=0 with 40 terms *)
series1 = Normal[Series[Exp[z], {z, 0, 40}]];

(* Plotting magnitude of function vs magnitude of series on the
   real axis *)
Plot[{Abs[Exp[x]], Abs[series1 /. z -> x]}, {x, -5, 5},
 PlotLegends -> {"|Exp[x]|", "|Series|"},
 PlotLabel -> "Taylor Expansion of Exp[z] (n=40)"]

(* Part (ii): Exp[z^2] around z=0 with 160 terms *)
series2 = Normal[Series[Exp[z^2], {z, 0, 160}]];
```

Experiment 8: Determining Required Terms for Accuracy

Objective: Determine how many terms should be used in the Taylor series expansion of $\exp(z)$ around $z = 0$ to get a percentage error $< 5\%$ for (i) $z = 30 + 30i$ (ii) $z = 10 + 103i$.

Mathematica Code:

```
(* Function to calculate percentage error for n terms at a given
   z-value *)
ErrorPercent[n_, zval_] :=
 Abs[(Exp[zval] - Normal[Series[Exp[z], {z, 0, n}]] /. z -> zval)
     / Exp[zval]] * 100
```

```

(* Part (i): Find necessary terms for z = 30 + 30I *)
terms1 = SelectFirst[Range[1, 200], ErrorPercent[#, 30 + 30*I] <
   5 &]

(* Part (ii): Find necessary terms for z = 10 + 103I *)
terms2 = SelectFirst[Range[1, 400], ErrorPercent[#, 10 + 103*I] <
    5 &]
```

Experiment 9: Laurent Series Expansion

Objective: To perform Laurent series expansion of a given function around $z = 0$. (i) $f(z) = (\sin z - 1)/z^4$ (ii) $f(z) = \cot(z)/z^4$.

Mathematica Code:

```
(* Part (i): Laurent Series of (Sin[z] - 1)/z^4 *)
laurent1 = Series[(Sin[z] - 1)/z^4, {z, 0, 5}]

(* Part (ii): Laurent Series of Cot[z]/z^4 *)
laurent2 = Series[Cot[z]/z^4, {z, 0, 5}]
```