🧮 ODE PRACTICAL MANUAL

Course Title: Ordinary Differential Equations
Couse Code: MAT-3104C
Software Used: Mathematica

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🧮 Practical 1: Plotting of Second-Order Solution Family of a Differential Equation

🎯 Objective

To obtain and plot the family of solutions for a second-order differential equation.

📘 Mathematical Model

$$y'' + y = 0$$

$$\Rightarrow y(x) = C_1 \cos(x) + C_2 \sin(x)$$

💻 Mathematica Code

```
sol = DSolve[y''[x] + y[x] == 0, y[x], x];
y[x_] = y[x] /. First[sol];

Plot[
 Evaluate[Table[y[x] /. {C[1] -> a, C[2] -> b}, {a, -2, 2, 1}, {b,
-2, 2, 1}]],
 {x, -10, 10},
 PlotRange -> All,
 AxesLabel -> {"x", "y(x)"},
 PlotLabel -> "Family of Solutions for y'' + y = 0",
 PlotStyle -> Table[ColorData["Rainbow"][i/9], {i, 1, 9}]
]
```

📈 Interpretation

Different curves correspond to different constants C_1 and C_2, forming the family of sinusoidal solutions.

🧮 Practical 2: Plotting of Third-Order Solution Family of a Differential Equation

🎯 Objective

To find and visualize the family of solutions of a third-order ODE.

📘 Mathematical Model

$$y''' - y = 0$$

$$\Rightarrow y(x) = C_1 e^x + e^{-\frac{x}{2}}\left[C_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + C_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right]$$

💻 Mathematica Code

```
sol = DSolve[y'''[x] - y[x] == 0, y[x], x];
y[x_] = y[x] /. First[sol];

Plot[
 Evaluate[
  Table[y[x] /. {C[1] -> a, C[2] -> b, C[3] -> c}, {a, -1, 1, 1},
{b, -1, 1, 1}, {c, -1, 1, 1}]
 ],
 {x, -5, 5},
 AxesLabel -> {"x", "y(x)"},
 PlotLabel -> "Family of Solutions for y''' - y = 0"
]
```

📈 Interpretation

A family of exponential and oscillatory curves showing third-order behavior.

🧮 Practical 3: Growth Model (Exponential Case)

🎯 Objective

To model population growth assuming continuous exponential increase.

📘 Mathematical Model

$$\frac{dP}{dt} = kP$$

$$\Rightarrow P(t) = P_0 e^{kt}$$

💻 Mathematica Code

```
k = 0.3; P0 = 100;
P[t_] = P0 Exp[k t];

Plot[P[t], {t, 0, 10},
 AxesLabel -> {"t", "P(t)"},
 PlotLabel -> "Exponential Growth Model",
 PlotStyle -> Thick]
```

📈 Interpretation

Shows population increasing exponentially with growth rate k.

🧮 Practical 4: Decay Model (Exponential Case)

🎯 Objective

To study exponential decay process (radioactive or chemical).

📘 Mathematical Model

$$\frac{dN}{dt} = -kN$$

$$\Rightarrow N(t) = N_0 e^{-kt}$$

💻 Mathematica Code

```
k = 0.2; N0 = 100;
N[t_] = N0 Exp[-k t];

Plot[N[t], {t, 0, 20},
 AxesLabel -> {"t", "N(t)"},
 PlotLabel -> "Exponential Decay Model",
 PlotStyle -> Thick]
```

📈 Interpretation

Depicts exponential decrease of quantity with time due to decay.

🧮 Practical 5: Lake Pollution Model

🎯 Objective

To model pollution concentration in a lake with inflow/outflow effects.

📘 Mathematical Model

$$\frac{dP}{dt} = r(C - P)$$

$$\Rightarrow P(t) = C + (P_0 - C)e^{-rt}$$

💻 Mathematica Code

```
r = 0.1; C = 50; P0 = 200;
P[t_] = C + (P0 - C) Exp[-r t];

Plot[P[t], {t, 0, 100},
 AxesLabel -> {"t", "P(t)"},
 PlotLabel -> "Lake Pollution Model",
 PlotStyle -> Thick]
```

📈 Interpretation

Pollution concentration gradually approaches equilibrium value C.

🧮 Practical 6: Drug Assimilation Model (Single Cold Pill)

🎯 Objective

To model drug concentration in the bloodstream over time.

📘 Mathematical Model

$$\frac{dC}{dt} = I - kC$$

$$\Rightarrow C(t) = \left(\frac{I}{k}\right)(1 - e^{-kt})$$

💻 Mathematica Code

```
I = 5; k = 0.4;
C[t_] = (I/k) (1 - Exp[-k t]);

Plot[C[t], {t, 0, 20},
 AxesLabel -> {"t", "C(t)"},
 PlotLabel -> "Drug Assimilation (Single Dose)",
 PlotStyle -> Thick]
```

📈 Interpretation

Concentration rises rapidly at first and levels off as elimination balances intake.

🧮 Practical 7: Limited Growth of Population (With and Without Harvesting)

🎯 Objective

To study population growth with natural limitation and harvesting effect.

📘 Mathematical Model (A) Without Harvesting

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)\ B)$$

💻 Mathematica Code (A)

```
r = 0.2; K = 500; P0 = 50;
P[t_] = K / (1 + ((K - P0)/P0) Exp[-r t]);

Plot[P[t], {t, 0, 50},
 AxesLabel -> {"t", "P(t)"},
 PlotLabel -> "Logistic Growth (No Harvesting)",
 PlotStyle -> Thick]
```

📘 Mathematical Model (B) With Harvesting

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - hP$$

💻 Mathematica Code (B)

```
r = 0.2; K = 500; h = 0.05; P0 = 50;
sol = DSolve[{P'[t] == r P[t] (1 - P[t]/K) - h P[t], P[0] == P0},
P[t], t];
Plot[Evaluate[P[t] /. sol], {t, 0, 100},
 AxesLabel -> {"t", "P(t)"},
 PlotLabel -> "Limited Growth with Harvesting",
 PlotStyle -> Thick]
```

📈 Interpretation

The population stabilizes at a lower equilibrium when harvesting is introduced compared to the natural logistic case.
